# HYPERSONIC FLOW AROUND A CIRCLE AND A SPHERE IN A MAGNETIC FIELD

**(GIPERZVUKOVOE TECHENIE OKOLO KRUGA I SFERY V MAGNITNOM POLE)**

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P.I. CHUSHKIN
(Moscow)

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When blunt bodies move with highly supersonic velocities strong ionization of the air occurs behind the detached shock wave. With the help of an external magnetic field, therefore, it is possible to influence this flow so as to cause a decrease in the heat transfer and an increase in the drag of the body, which is important for flying machines entering the dense layers of the atmosphere. A case of practical interest is such a flow at moderate magnetic Reynolds numbers $R_m$; the field of flow does not then have any influence on the magnetic field, which can therefore be regarded as specified. In [1-3] hypersonic flow was considered in the neighborhood of the stagnation point of a magnetized sphere, when the density behind the shock wave was assumed constant and the number $R_m$ small. Bush's investigation [4], which studied the analogous problem for finite values of the magnetic Reynolds' number, showed that its influence on the solution was weak.

Below we calculate the flow around a circle or a sphere (in all its region of influence), moving with highly supersonic velocity in a magnetic field at small values of $R_m$. For the solution we applied Dorodnitsyn's method of integral relations [5]. Belotserkovskii [6,7] used this method to solve the problem of supersonic flow past blunt bodies in a nonconducting gas. We give below a generalization of this solution to the case of magnetized bodies in a conducting gas.

**1. Formulation and equations of the problem.** Suppose that a blunt magnetized body of given shape (circle or sphere) is placed in a uniform stream of perfectly inviscid, thermally nonconducting gas flowing along its axis of symmetry with a highly supersonic value of the Mach number $M_\infty$. The gas in front of the detached shock wave is assumed to be nonconducting electrically, whilst behind the shock wave the coefficient of electrical conductivity $\eta$ of the gas is considered to be either constant, or proportional to the fifth power of the temperature. The magnetic Reynolds' number is assumed to be small. It is assumed that the magnetic field is created by a plane or axisymmetric magnetic dipole, located at the center of the circle or sphere, respectively, and having its axis parallel to the direction of the incident stream. The electric field is everywhere equal to zero.

The problem consists in determining the composite flow in the region of influence, and in particular in the construction of the detached shock wave and the calculation of the flow parameters on the surface of the body. Because of symmetry it is sufficient to consider only the upper half-plane.

The system of equations of magneto-gas-dynamics (the equation of continuity, the momentum equation and the entropy equation) in the case under consideration have the form

$$\operatorname{div} \rho \mathbf{w} = 0, \qquad \rho \frac{d\mathbf{w}}{dt} + \nabla p = \mathbf{F}, \qquad \frac{d\varphi}{dt} = -\frac{\varkappa - 1}{\rho^{\varkappa}} \mathbf{F}\cdot\mathbf{w} \tag{1.1}$$

where the ponderomotive force $\mathbf{F}$ is given by

$$\mathbf{F} = b\,(\mathbf{w}\times\mathbf{H})\times\mathbf{H}, \qquad b = \frac{\eta_* H_*^2 R}{c^2 \rho_\infty a_*^2} \tag{1.2}$$

System (1.1) is written in dimensionless variables, with the linear dimensions referred to the radius of the body $R$, the velocity $w$ to the critical velocity of sound $a_*$, the density $\rho$ to the density of the incident stream $\rho_\infty$, the pressure $p$ to the quantity $\rho_\infty a_*^2$ and the intensity of the magnetic field $H$ to its value $H_*$ at the stagnation point. Moreover, in equations (1.1) and (1.2) $\kappa$ denotes the adiabatic index, $\varphi$ the entropy function, $b$ the dimensionless parameter characterizing the strength of the magnetic field, $c$ the velocity of light and $\eta_*$ the value of the coefficient of electrical conductivity at the stagnation point

$$\varphi = p / \rho^{\varkappa}, \qquad S = c_v \ln \varphi + \text{const}$$

where $S$ is the entropy and $c_v$ is the coefficient of thermal capacity at constant volume.

The solution of the problem will be developed for the case of a circle ($\nu = 0$) and a sphere ($\nu = 1$); we shall therefore use polar coordinates $r$ and $\theta$ with the pole at the center of the body. The components of the magnetic field vector, created by a dipole placed at the center of the circle or the sphere, are equal to

$$H_r = \frac{\cos\theta}{r^{2+\nu}}, \qquad H_\theta = \frac{\sin\theta}{(1+\nu)\, r^{2+\nu}}$$

The corresponding components of the ponderomotive force are expressed thus:

$$F_r = -b\,(uH_\theta - vH_r)\,H_\theta, \qquad F_\theta = b\,(uH_\theta - vH_r)\,H_r$$

where $u$ and $v$ are the components of the velocity vector $\mathbf{w}$ along $r$ and $\theta$, respectively.

The three equations of system (1.1) give the variation of entropy along a streamline conditioned by the action of the magnetic field, and can be rewritten in the form

$$d\varphi = -\frac{\varkappa - 1}{\rho^{\varkappa}}\,(F_r\, dr + F_\theta\, r\, d\theta), \qquad \frac{dr}{r d\theta} = \frac{u}{v} \tag{1.3}$$

The system of equations (1.1) has a Bernoulli integral, which in the case under consideration in the absence of electric field is given by the expression

$$p = \frac{\varkappa + 1}{2\varkappa}\left(1 - \frac{\varkappa - 1}{\varkappa + 1}\, w^2\right)\rho \tag{1.4}$$

The Bernoulli integral will take the place of the momentum equation along the curve $r$ = const. Let us introduce the function $\tau$

$$\tau = \rho\varphi^{-\frac{1}{\varkappa-1}} = \left[\frac{\varkappa+1}{2\varkappa}\left(1 - \frac{\varkappa-1}{\varkappa+1}\, w^2\right)\right]^{\frac{1}{\varkappa-1}}$$

and transform the equation of continuity by taking account of (1.3). The transformed equation of continuity and the equation of momentum along $\theta$ = const will be written in the form

$$\frac{\partial rh}{\partial r} + \frac{1}{\mu}\frac{\partial \mu q}{\partial\theta} = G, \qquad \frac{\partial rQ}{\partial r} + \frac{1}{\mu}\frac{\partial \mu P}{\partial\theta} = g \tag{1.5}$$

where

$$h = r^{\nu}\tau u, \qquad q = r^{\nu}\tau v, \qquad \mu = \sin^{\nu}\theta, \quad G = -r^{1+\nu}\tau p^{-1}\,(\mathbf{F}\cdot\mathbf{w})$$

$$Q = r^{\nu}\,(p + \rho u^2), \qquad P = r^{\nu}\,\rho u v, \qquad g = r^{\nu}\,(2^{\nu} p + \rho v^2 + rF_r)$$

Let us denote by $\varepsilon$ the distance along a ray $\theta$ = const from the contour of the body to the shock wave and by $\sigma$ the angle of inclination of the shock wave to the axis of the flow. Then, to determine the radius of the shock wave

$$r_n(\theta) = 1 + \varepsilon(\theta)$$

we have the equation

$$\frac{d\varepsilon}{d\theta} = -(1+\varepsilon)\cot(\sigma+\theta) \tag{1.6}$$

Equations (1.3) to (1.6) form a system for the five unknown functions $u$, $v$, $p$, $\varphi$ and $\sigma$. The boundary conditions for this system are as follows:

on the body surface ($r = 1$)

$$u = 0$$

on the shock wave

$$u_n = w_y \sin\theta - w_x \cos\theta, \qquad v_n = w_x \sin\theta + w_y \cos\theta$$

$$w_x = w_\infty\left[1 - \frac{2}{\varkappa+1}\left(\sin^2\sigma - \frac{1}{M_\infty^2}\right)\right], \qquad w_y = (w_\infty - w_x)\cot\sigma \tag{1.7}$$

$$w_\infty^2 = \frac{(\varkappa+1)M_\infty^2}{2+(\varkappa-1)M_\infty^2}, \qquad p_n = \frac{2}{\varkappa+1}w_\infty^2\sin^2\sigma - \frac{\varkappa-1}{2\varkappa}\left(1-\frac{\varkappa-1}{\varkappa+1}w_\infty^2\right)$$

$$\rho_n = \frac{(\varkappa+1)w_\infty^2\sin^2\sigma}{(\varkappa+1)-(\varkappa-1)w_\infty^2\cos^2\sigma}, \qquad \varphi_n = \frac{p_n}{\rho_n^{\varkappa}}$$

on the flow axis ($\theta = 0$)

$$v = 0, \qquad \sigma = \frac{\pi}{2}, \qquad \varphi = \varphi_* = \frac{\varkappa+1}{2\varkappa}w_\infty^{-2\varkappa}\left(w_\infty^2 - \frac{\varkappa-1}{\varkappa+1}\right) \tag{1.8}$$

Here the last equation follows from the shock wave relations with $\sigma = 1/2\,\pi$ and equation (1.3), which shows that behind the shock wave on the axis $\theta = 0$ variation of entropy does not occur, since $F_r = 0$.

**2. Integral relations and the approximating system.** For the solution of system (1.3) to (1.6) we shall apply the method of integral relations. Considering the $n$th approximation, the region of flow between the body and the shock wave is divided into $n$ zones by means of the system of curves

$$r = r_i(\theta) = 1 + \frac{i}{n}\varepsilon(\theta) \qquad (i = 0, \ldots, n)$$

The values of all the functions on the curve $r = r_i(\theta)$ will be denoted by the subscript $i$, so that the body contour will correspond to the subscript 0, and the shock wave to the subscript $n$. We shall integrate the partial differential equations (1.5) with respect to $r$ from $r_0 = 1$ up to each curve $r = r_i(\theta)$ ($i = 1, \ldots, n$). Then taking account of the fact that $h_0 = 0$ we obtain a system of $2n$ independent integral relations

$$\frac{1}{\mu}\frac{d}{d\theta}\int_1^{r_i}\mu q\,dr - q_i\frac{dr_i}{d\theta} + r_i h_i = \int_1^{r_i} G\,dr$$

$$\frac{1}{\mu}\frac{d}{d\theta}\int_1^{r_i}\mu P\,dr - P_i\frac{dr_i}{d\theta} + r_i Q_i - Q_0 = \int_1^{r_i} g\,dr \qquad (i = 1, \ldots, n)$$

All the integrands are represented approximately by interpolation polynomials of the $n$th degree in $r$ with centers of interpolation on the curves $r = r_i(\theta)$. As a result the integral relations reduce to $2n$ ordinary differential equations with respect to $\theta$ for the values of the required functions on the curves $r = r_i(\theta)$. Including equations (1.3) and (1.6), the whole approximating system will include $3n + 1$ differential equations and, after solution, can be represented in the following form:

$$\frac{d\varepsilon}{d\theta} = -(1+\varepsilon)\cot(\sigma+\theta), \qquad \frac{d\sigma}{d\theta} = \frac{E_\sigma}{D_0}, \qquad \frac{dv_i}{d\theta} = \frac{E_i}{D_i}$$
$$\frac{d\varphi_i}{d\theta} = -\frac{\varkappa-1}{\rho_i^{\varkappa}}\left(F_i\frac{u_i}{v_i} + r_i F_{\theta i}\right), \qquad D_i = \frac{2}{\varkappa+1}(a_i^2 - v_i^2) \tag{2.1}$$
$$a_i^2 = \varkappa\frac{p_i}{\rho_i}, \qquad \frac{du_i}{d\theta} = U_i \qquad (u_0 \equiv 0) \qquad (i = 0, \ldots, n-1)$$

where $E_\sigma$, $E_i$ and $U_i$ are certain functions, holomorphic in the region under consideration, with forms depending upon $n$. The denominators $D_i$ vanish at the points where the velocity component $v_i$ attains the local speed of sound $a_i$, i.e. at the points where the ray $\theta$ = const touches the characteristic. This connection between the characteristics and the singular points is essential in the given method. Accordingly, the $n$ equations of system (2.1) have a moving singular point which, as investigation shows, is a saddle point.

The approximating system (2.1) is integrated numerically from the axis $\theta = 0$, where conditions (1.8) apply to the functions $v_i$, $\sigma$ and $\varphi_i$. The unknowns here, i.e. the $n - 1$ values of $u_i$ and the value of $\varepsilon$, are determined by the requirement for regularity of the solution at the $n$ singular points, where we must have $E_i = 0$ (and automatically $E_\sigma = 0$). After solution of this boundary problem the values of the required functions are found on the curves $r = r_i(\theta)$, and from them, to the degree of approximation assumed, the whole field of flow can be constructed. We notice that in the axisymmetric case the point $\theta = 0$ for system (2.1) is also a singular point, but of regular type, and the indeterminacy here is easily elucidated.

We shall now derive the actual form of the approximating system (2.1) for the approximation $n = 1$. In this case

$$\frac{d\sigma}{d\theta}=\frac{1}{\alpha_1}\left[g_0+g_1+(Q_0-r_1Q_1)\frac{2}{\varepsilon}-r_1{}^{\nu}\rho_1(v_1{}^2-u_1{}^2)+\frac{P_1}{r^{\nu}{}_1\varepsilon}\frac{d\varepsilon}{d\theta}-\delta_1\right]$$
$$\frac{dv_0}{d\theta}=\frac{2\varkappa\tau_0{}^{\varkappa-2}}{(\varkappa+1)D_0}\left[G_0+G_1+\left(1-\frac{2r_1}{\varepsilon}\right)h_1-\left(q_0-\frac{q_1}{r_1}\right)\frac{1}{\varepsilon}\frac{d\varepsilon}{d\theta}-\alpha_0\frac{d\sigma}{d\theta}-\delta_0\right] \qquad (2.2)$$
$$\frac{d\varphi_0}{d\theta}=-\frac{\varkappa-1}{\rho_0{}^{\varkappa}}F_{\theta 0},\qquad \frac{d\varepsilon}{d\theta}=-r_1\cot(\sigma+\theta)$$

where

$$\alpha_0=\frac{r_1{}^{\nu}\tau_1{}^{2-\varkappa}}{2\varkappa}\left[(\varkappa+1)(1-v_1{}^2)v_1'-2u_1v_1u_1'\right]$$
$$\alpha_1=\frac{\rho_1}{\tau_1}\left[u_1\left(\alpha_0-\frac{q_1}{\varkappa-1}\varphi_1'\right)+q_1u'_1\right]$$
$$\delta_0=\nu(q_0+q_1)\cot\theta,\qquad \delta_1=\nu P_1\cot\theta$$

and $u_1'$, $v_1'$ and $\varphi_1'$ here signify the derivatives with respect to $\sigma$ of these functions at the shock wave, which are determined from formulas (1.7).

In equations (2.2) for $\sigma$ and $v_0$ in the axisymmetric case when $\theta = 0$ there is indeterminacy in the terms $\delta_0$ and $\delta_1$. After resolving the indeterminacy, the equations for $\sigma$ and $v_0$ may be put into the previous form, but in them it is necessary to double the values for $D_0$, $\alpha_0$ and $\alpha_1$ and set

$$\delta_0=-h_1,\qquad \delta_1=-r_1\rho_1u_1{}^2.$$

We notice again that when $n = 1$ the equation for $v_0$, when $\theta = 0$ and with the given $M_\infty$, $\kappa$ and $b$, connects the velocity gradient at the stagnation point $dv_0/d\theta$ with the distance of detachment of the shock wave $\varepsilon$. Insofar as the latter quantity has been determined by experiments in shock tubes, then from $\varepsilon$ by means of this connection we can find approximately the value of $dv_0/d\theta$, which plays an important role in the calculation of heat transfer.

**3. Examples.** Using the method described above to the first approximation ($n = 1$) we have calculated the flow past a magnetized circle and sphere for highly supersonic velocities. The calculations were carried out only for the region of subsonic and mixed flow, i.e. in the cases considered up to values of $\theta$ corresponding to the sonic point on the body. Generally speaking, by the method of integral relations we can calculate the supersonic flow also up to large values of $\theta$, but here it is more convenient to apply the numerical method of characteristics.

In Figs. 1 to 3 are shown the results of the calculations for the case of flow past a sphere in a stream of air ($\kappa$ = 1.4) with Mach number $M_\infty = \infty$, for values of the magnetic field parameter $b$ = 0, 4 and 16. For the case $b$ = 16, in order to show the nature of the influence

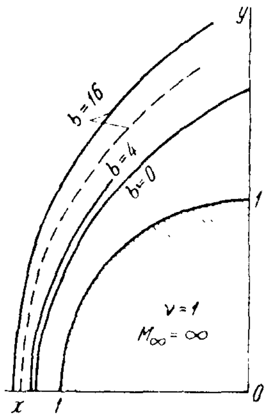


Fig. 1.

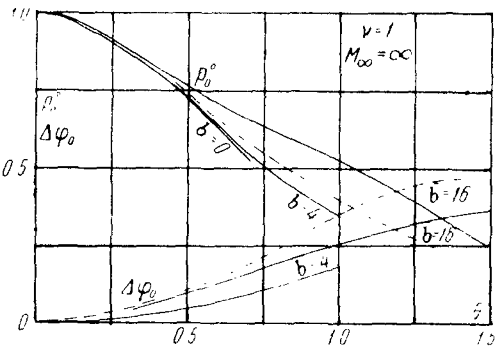


Fig. 2.

of the coefficient of electrical conductivity $\eta$, we considered two variants - in one (the continuous curves in the pictures) the quantity $\eta$ was assumed constant, whilst in the other (the dotted curves) we assumed the following dependence of $\eta$ on the temperature $T$:

$$\frac{\eta}{\eta_*} = \left(\frac{T}{T_*}\right)^5 = \left(1 - \frac{\kappa - 1}{\kappa + 1}\, w^2\right)^5$$

where $\eta_*$ and $T_*$ are the values of $\eta$ and $T$ at the stagnation point. This functional dependence was obtained by processing the experimental data of [8].

In Fig. 1 are shown the shock waves for $b$ = 0, 4 and 16. In all these cases the sonic line is situated below the ray $\theta$ = const through the sonic point on the body.

The influence of the magnetic field on the pressure distribution on the body $p_0^o = p_0/p_*$, expressed in terms of the pressure at the stagnation point $p_*$, is shown in Fig. 2. Stronger magnetic fields cause a significant increase in the pressure on the body and a pronounced movement of the sonic point towards the direction of increasing $\theta$, but in the case of variable $\eta$ these changes are appreciably smaller. The calculations show that for the sphere the coefficient of wave drag $c_x$ for the portion of the body bounded by the sonic point increases in the case $\eta$ = const when $b$ = 4 by 2.5 per cent and when $b$ = 16 by 24 per cent as compared with $c_x$ for the same region of the body when $b$ = 0. In the case of variable $\eta$, however, when $b$ = 16 the corresponding increase in $c_x$ amounts to 8.8 per cent. For orientation purposes we notice that in the conditions of orbital flight of a sputnik the value of the parameter $b = 1$ corresponds to a magnetic field intensity $H_*$ of order $10^4$ gauss.

In Fig. 2 are shown also the curves of variation of the entropy function $\Delta\varphi_0 = (\varphi_0 - \varphi_*)/\varphi_*$ along the contour of the body.

In Fig. 3 we show as a function of the parameter $b$, for a sphere at $M_\infty = \infty$ and $\kappa = 1.4$, the ratio $\varepsilon^\circ$ of the distance of detachment of the shock wave at the stagnation point to the corresponding distance of detachment of the shock wave in the absence of a magnetic field ($b = 0$). Here also is shown the analogous ratio for the gradient of velocity at the stagnation point on the body $(v_0')^\circ$.

Finally, in Fig. 4 we present the results showing the influence of the Mach number $M_\infty$. Here are shown the pressure distribution $p_0^\circ$ and the entropy function $\Delta\varphi_0$

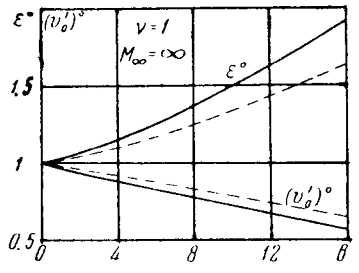


Fig. 3

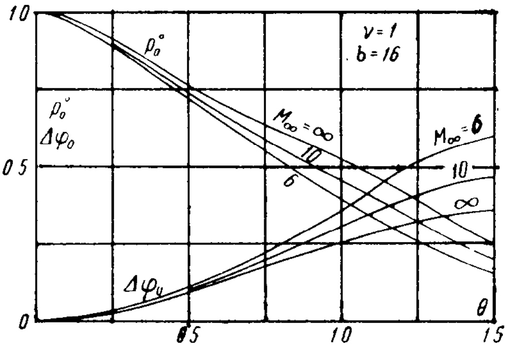


Fig. 4.

on the sphere for a value of the parameter $b = 16$ and $M_\infty = 6$, 10 and $\infty$. We notice that the quantities $\varepsilon^\circ$ and $(v_0')^\circ$ in this case depend very weakly on $M_\infty$.

In conclusion we observe that the method of solution, described in this paper only for the case of the circle and the sphere, can be extended without difficulty to the case of blunt symmetric bodies of more general shape. Then instead of the polar coordinates $r$ and $\theta$ it is appropriate to use coordinates $s_0$ and $n_0$, where $s_0$ is arc length measured along the contour of the body and $n_0$ is the normal to the contour of the body. We notice that for solution of the problem under consideration we can also apply another scheme, in which the approximation is carried out for $\theta$ whilst the approximating system is integrated with respect to $r$ from the shock wave to the body. Then the form of the shock wave is determined by fulfilling the condition of no flow across the body.

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